# Design Rainfall Temporal Distributions for 24 Hour Rainfall Events Having Average Return Intervals of 1 to 100 Years 

## Appendix B. Developing rainfall temporal distributions for small average return intervals.

prepared by
Philip H. De Groot, Ph.D., P.E.; Principal Hydraulic Engineer Michael C. Menoes, Ph.D., P.E.; Senior Hydraulic Engineer

Hydrosphere Engineering PO BOX 360530
Cleveland, Ohio 44136-0009
330-721-2722 or 440-879-2049
hydrosphere-engineering.com

## OBJECTIVE

Develop 24 hour synthetic rainfall temporal distributions that can accurately predict the peak rate of storm water runoff from rainfall events having average return periods from 1 to 100 years.

Step 1. Adjust time scale of the NRCS historical Type 2, 24 hour storm so that $50 \%$ of the rainfall depth occurs at exactly 12 hours.

Figure B1. NRCS Type 2, 24 hour rainfall temporal distribution.


Figure B2. Central 2 hour time period of the NRCS Type 2 rainfall temporal distribution with a time shift of 0.15 hours so that $50 \%$ of the depth occurs at 12 hours.


Step 2. Split the adjusted NRCS historical Type 2, 24 hour storm into two 12 hour storms. Examine the rainfall distribution from 12 hours to 24 hours.

Figure B3. Final 12 hours of the NRCS Type 2, 24 hour rainfall temporal distribution with the time shift so that $50 \%$ of the rainfall depth occurs at 12 hours.


Step 3. Use a form of an intensity-frequency-duration equation and the method of least squares to fit a curve to the final 12 hours of the time adjusted NRCS historical Type 2, 24 hour rainfall temporal distribution

Intensity-frequency-duration equation form for the prediction of the depth of rainfall

$$
d_{P}=0.5 *\left\{1+\left[\frac{a * t}{(b+t)^{c}}\right]\right\}
$$

where $t=$ time in hours from 0 to 12
$\mathrm{d}_{\mathrm{P}}=$ fraction of rainfall depth
$a, b, c=$ coefficients determined from the least squares analysis

Objective function:

$$
\text { Minimize } \sum_{i=1}^{6}\left(d_{P i}-d_{O i}\right)^{2}
$$

Constraints:
(6 equations: $\mathrm{i}=1$ to 6 )

$$
d_{P i}=0.5 *\left\{1+\left[\frac{a * t_{i}}{\left(b+t_{i}\right)^{c}}\right]\right\}
$$

where: $d_{O i}=$
observed fraction of the rainfall taken from the table of the adjusted NCRS Type 2, 24 hour rainfall temporal distribution
$d_{P i}=\quad \begin{gathered}\text { predicted fraction of the rainfall determined from the least } \\ \text { squares analysis }\end{gathered}$
$t_{i}=\quad$ time in hours

Values of $\mathrm{t}_{\mathrm{i}}$ and $\mathrm{d}_{\mathrm{O}}$ used to a fit the curve:

|  | Time since <br> beginning <br> of the storm <br> (hours) | Time used <br> to fit the <br> curve <br> (hours) <br> $\mathrm{t}_{\mathrm{i}}$ values | Fraction <br> of the <br> total <br> rainfall |
| :---: | :---: | :---: | :---: |
| $\mathrm{d}_{\mathrm{Oi}}$ values |  |  |  |$|$| 1 | 12.00 | 0.0000001 | 0.500 |
| :--- | :--- | :--- | :--- |
| 2 | 12.65 | 0.65 | 0.735 |
| 3 | 13.65 | 1.65 | 0.799 |
| 4 | 18.15 | 6.15 | 0.921 |
| 5 | 21.15 | 9.15 | 0.965 |
| 6 | 24.00 | 12.00 | 1.000 |

Figure B4. Final 12 hours of the adjusted NRCS Type 2, 24 hour rainfall temporal distribution and the fitted curve for the same distribution.


Step 4. Use the fitted intensity-frequency-duration equation for the initial 12 hours of the rainfall event to complete the rainfall temporal distribution for the entire 24 hours.

Equation that is applicable to the first 12 hours of the 24 hour temporal distribution:

$$
d_{P}=0.5 *\left\{1-\left[\frac{a^{*} t}{(b+t)^{c}}\right]\right\}
$$

where the variables were previously defined.

Figure B5. Original NRCS Type 2, 24 hour rainfall temporal distribution and the curve fitted to match the same distribution.


Step 5. Develop a family of rainfall temporal distributions that are less severe than the NCRS Type 2, 24 hour event.

The results of the least squares analysis yielded values of $a, b$ and $c$ that, when inserted into the equation and then used in a hydrologic model, produce a peak rate of storm water runoff slightly higher than that produced by the historical NRCS Type 2, 24 hour rainfall temporal distribution. A family of rainfall curves can now developed that will produce lower values of the peak rate of storm water runoff. The results of the initial least squares analysis are identified as:

| Name | a | b | $c$ |
| :--- | :--- | :--- | :--- |
| HY_00 | 0.5261 | $1.423 E-3$ | 0.7418 |

A 24 hour rainfall temporal distribution that produces a minimum peak rate of storm water runoff has a steady rainfall over the entire 24 hours. If the HY_00 rainfall temporal distribution is plotted on the same axes as a uniform rainfall temporal distribution, the vertical distance between the curves is an indication of the severity of the rainfall temporal distribution. The slope of the curve at any point is the rainfall intensity. The rainfall temporal distributions which produce the highest rate of storm water runoff have the steepest slopes. Figure B6 shows a plot of HY_00, a uniform temporal distribution, and a series of curves. The HY_00 is the most severe, and the severity gradually decreases for each curve moving downward toward the uniform temporal distribution.

Figure B6. Developing intermediate rainfall temporal distributions by fitting curves between HY_00 and a uniform rainfall temporal distribution.


A least squares analysis was performed for a series of 10 intermediate curves between HY_00 and the uniform rainfall distribution. Table B1 shows the fractional values of the rainfall at the various times, and the resultant coefficients for each intermediate curve.

Table B1. Time and fractional values of the rainfall depth that were used to determine the coefficients $a, b$ and $c$ for each of the rainfall temporal distributions.

| Time | hours | 12.00 | 12.65 | 13.65 | 18.15 | 21.15 | 24 | a | b | c |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Version | HY_00 | 0.5 | 0.735 | 0.799 | 0.921 | 0.965 | 1 | 0.5261 | $1.423 \mathrm{E}-03$ | 0.7418 |
|  | HY_01 | 0.5 | 0.720 | 0.782 | 0.909 | 0.959 | 1 | 0.4916 | $1.27 \mathrm{E}-05$ | 0.7166 |
|  | HY_02 | 0.5 | 0.705 | 0.765 | 0.897 | 0.953 | 1 | 0.4576 | $7.092 \mathrm{E}-06$ | 0.6896 |
|  | HY 03 | 0.5 | 0.690 | 0.748 | 0.885 | 0.947 | 1 | 0.4234 | $4.244 \mathrm{E}-06$ | 0.6600 |
|  | HY_04 | 0.5 | 0.675 | 0.731 | 0.873 | 0.941 | 1 | 0.3891 | $2.499 \mathrm{E}-06$ | 0.6274 |
|  | HY_05 | 0.5 | 0.660 | 0.714 | 0.861 | 0.935 | 1 | 0.3547 | $1.386 \mathrm{E}-06$ | 0.5911 |
|  | HY_06 | 0.5 | 0.645 | 0.697 | 0.849 | 0.929 | 1 | 0.3202 | $6.877 \mathrm{E}-07$ | 0.5507 |
|  | HY_07 | 0.5 | 0.630 | 0.680 | 0.837 | 0.923 | 1 | 0.2858 | $2.74 \mathrm{E}-07$ | 0.5053 |
|  | HY_08 | 0.5 | 0.615 | 0.663 | 0.825 | 0.917 | 1 | 0.2516 | $5.197 \mathrm{E}-08$ | 0.4539 |
|  | HY_09 | 0.5 | 0.600 | 0.646 | 0.813 | 0.911 | 1 | 0.2179 | $-5.059 \mathrm{E}-08$ | 0.3954 |
|  | HY_10 | 0.5 | 0.585 | 0.629 | 0.801 | 0.905 | 1 | 0.1851 | $-8.837 \mathrm{E}-08$ | 0.3286 |

Figure B 6 shows a plot of the completed family of rainfall temporal distributions with HY_00 being the most severe and HY_10 being the least severe.

Figure B6. Family of 24 hour symmetric rainfall temporal distribution curves with $50 \%$ of the rainfall depth occurring at 12 hours. HY_00 produces the most runoff, HY_10 the least runoff.


